An N-Way Hybrid Power Divider*

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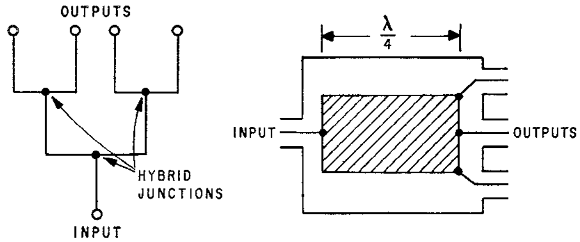
Summary—**A circularly symmetric power divider is described which splits a signal into *n* equiphase equiamplitude parts where *n* can be odd or even. The power divider provides isolation between output terminals and approximately matched terminal impedances over about a 20 per cent band. A theory of operation is given which yields the necessary design parameters, and an experimental model is described which has a minimum isolation of −27 db between output terminals, an output VSWR of 1.6, and an input VSWR of 1.2.**

Introduction

The problem of dividing up an input signal into a number of equiphase equiamplitude output signals is a familiar one to RF engineers, particularly those working in the field of phased arrays. It is often desired that the equiphase equiamplitude condition be obtained in a manner which is fundamentally independent of frequency, and, in addition, that a fairly high degree of isolation exist between output terminals over some specified frequency band. Two of the more commonly used power dividers are illustrated in Fig. 1. In Fig. 1(a), a corporate feed structure of T junctions provides the symmetry necessary to preserve the equiphase equiamplitude condition independent of frequency. In Fig. 1(b), circular symmetry is employed where all the output terminals are connected to the common center conductor of a coaxial line, and the combined load is matched by means of a quarter-wave transformer. To provide isolation between output terminals, the T junctions of Fig. 1(a) may be replaced by hybrid junctions. In addition to the cost and complexity of providing a number of separate, yet identical, hybrids, the corporate feed structure has the limitation of providing only a binary number of outputs. If nine outputs were required, for example, a 16-to-1 power divider would have to be used with $\frac{7}{16}$ of the power output being dissipated in matched loads. Fig. 1(b), on the other hand, because of the circular symmetry, provides any number of outputs. It has, however, the serious disadvantage of having no isolation between outputs, as the output terminals are badly mismatched because all loads appear in parallel across any given output terminal. This paper describes a device which possesses the circular symmetry of Fig. 1(b), so that it maintains phase and amplitude equality between any number of outputs independent of frequency, but, in addition, provides isolated and matched outputs.

Theory of Operation

The power divider as sketched in Fig. 2 consists of a coaxial line in which the hollow inner conductor has been split into n splines of length $\lambda/4$. A shorting plate



(a) (b)

Fig. 1.

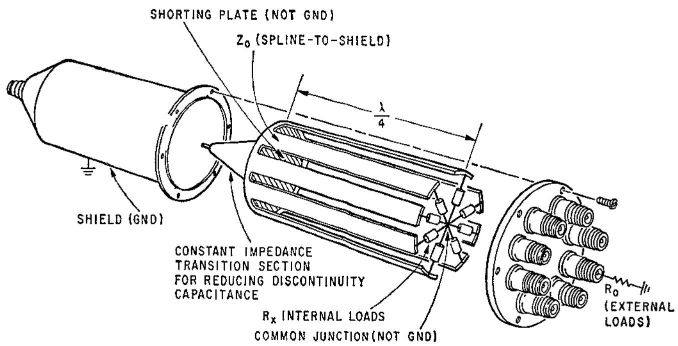


Fig. 2.

connects the splines at the input end,[1] and resistors are connected in a radial manner between each spline at the output end and a common junction. Output connectors are shown connected to the splines in an "in line" manner, although they may also be connected in radial fashion, *i.e.*, at right angles to the splines. When a signal is fed into the power divider, it divides by virtue of symmetry into n equiphase equiamplitude parts. No power is dissipated by the resistances when matched loads are connected to the outputs, since all splines will be at the same potential. However, if a reflection occurs at one of the output terminals, the reflected signal will split; part of it will travel directly to the remaining output terminals via the resistors, and the rest of it will travel back to the input, splitting again at the junction of the splines and then returning to the remaining output terminals. Thus, the reflected wave arrives at the remaining output terminals in two parts, and the path length difference between the two paths of travel will be 180° when the splines are $\lambda/4$ in length. It will be shown that when the value of the resistors and the characteristic impedance of the spline transmission lines are properly chosen, the two parts of the reflected wave are also equal in amplitude; hence, complete cancellation occurs.

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[1] A tapered section is shown at the input end for reducing the discontinuity caused by the abrupt change in line sizes when a standard type N connector is used. A more convenient form of this device, which eliminates the need for a separate tapered section, can be realized by uniformly tapering the splined coaxial section itself.

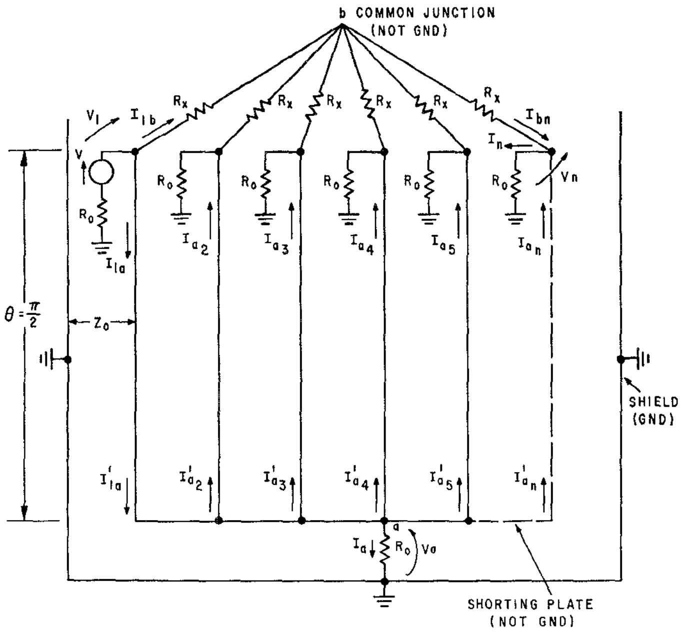

Fig. 3—R_0=external loads connected between splines and shield (GND); R_x=internal loads connected between splines and common junction point; Z_0=characteristic impedance of each spline-to-shield transmission line; V=voltage applied to a single output terminal; V_n=voltage appearing at nth output terminal; V_a=voltage appearing at input terminal.

Referring to Fig. 3, if a voltage V is applied to output terminal 1 by a generator of internal resistance R_0, then the voltages V_n appearing at the other output terminals must all be equal because of symmetry. Applying the transmission line equations when each spline transmission line is a quarter wavelength long ($\theta=\pi/2$), we have

$$\text{spline 1:}\quad V_1 = V_a \cos\theta + jI_{1a}'Z_0 \sin\theta = jI_{1a}'Z_0$$

$$I_{1a} = I_{1a}' \cos\theta + j\frac{V_a}{Z_0}\sin\theta = j\frac{V_a}{Z_0}$$

$$\text{spline } n\text{:}\quad V_a = V_n \cos\theta + jI_{an}Z_0 \sin\theta = jI_{an}Z_0 \tag{1}$$

$$I_{an}' = I_{an}\cos\theta + j\frac{V_n}{Z_0}\sin\theta = j\frac{V_n}{Z_0}.$$

It is also true that

$$I_{1a}' = \frac{V_a}{R_0} + (n-1)I_{an}'$$

$$I_{an} = \frac{V_n}{R_0} - \frac{I_{1b}}{n-1}$$

$$(I_{1b} + I_{1a})R_0 = V - V_1$$

$$\left(I_{1b} + \frac{I_{1b}}{n-1}\right)R_x = V_1 - V_n. \tag{2}$$

Combining (1) with (2) we obtain the following set of simultaneous equations for the unknown voltages V_1, V_a, V_n:

$$V_a + j(n-1)\frac{R_0}{Z_0}V_n + jV_1\frac{R_0}{Z_0} = 0$$

$$jV_a\frac{R_0}{Z_0} + \left(1 + \frac{R_0}{nR_x}\right)V_n - V_1\frac{R_0}{nR_x} = 0$$

$$jV_a\frac{R_0}{Z_0} - \left(\frac{n-1}{n}\right)\frac{R_0}{R_x}V_n + V_1\left(1 + \frac{R_0}{R_x}\left(\frac{n-1}{n}\right)\right) = V. \tag{3}$$

For perfect isolation $V_n=0$, (3) may be combined to yield

$$\frac{V_1}{V} = \frac{1}{1 + \dfrac{R_0}{R_x}}$$

$$\left(\frac{R_0}{Z_0}\right)^2 = \frac{R_0}{nR_x}, \tag{4}$$

and for matched output admittances, we obtain

$$Y_1 = \frac{I_{1b} + I_{1a}}{V_1} = \frac{V - V_1}{R_0V_1} = \frac{1}{R_0}. \tag{5}$$

Combining (4) and (5) yields

$$R = R_x$$

$$Z_0 = \sqrt{n}\,R_0. \tag{6}$$

Thus, if the internal loads R and the characteristic impedance of the spline-to-shield transmission lines are adjusted according to (6), the outputs will be completely isolated and matched. The input impedance under these conditions will be the parallel combination of the n output loads R_0, after each has been transformed through a quarter wavelength of line Z_0. Hence,

$$Z_{\text{input}} = \frac{\dfrac{Z_0^2}{R_0}}{n} = R_0$$

or, in other words, the input of the power divider is also matched when the conditions for isolation between outputs are satisfied.

Experimental Results

In order to determine whether or not the isolation and matched output features could be realized in a power divider of practical design, a power divider of the type illustrated in Fig. 1(b), was modified according to the following procedure. Narrow slots were milled in the hollow center conductor of an 8-to-1 power divider of this type to create eight spline-to-shield transmission lines having a characteristic inpedance of approximately eight times that of the unslotted coaxial line whose characteristic impedance was $R_0/\sqrt{8}$. Hence, the spline-to-shield lines had

$$Z_0 = 8\frac{R_0}{\sqrt{8}} = \sqrt{8}R_0$$

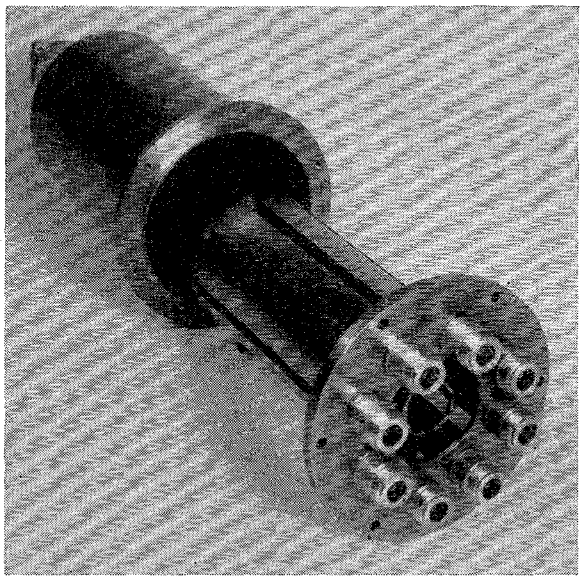

Fig. 4.

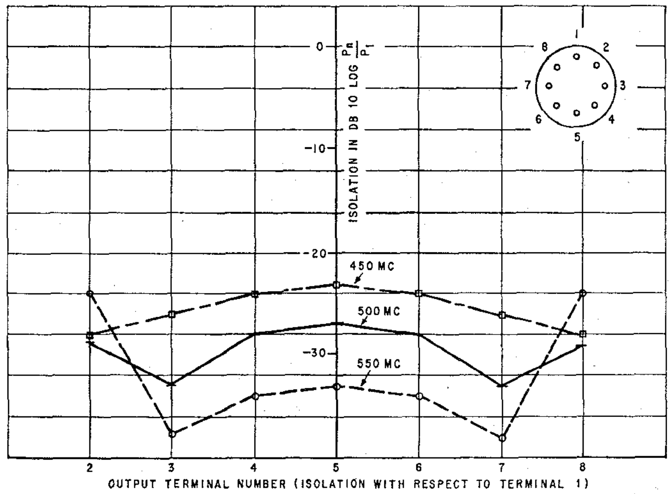

Fig. 5—Isolation between output terminals for N-way hybrid power divider.

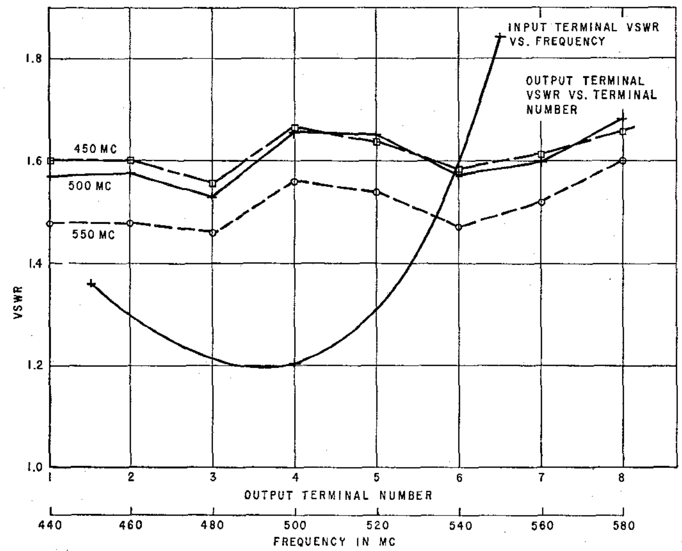

Fig. 6—VSWR characteristics for N-way hybrid power divider.

in accordance with (6). Eight resistors equal in value to R_0 (50 ohms) were then soldered to the ends of the splines, joining in a radial manner to a short circuiting ring (see Fig. 4). Isolation and output VSWR values were obtained using a parallel combination of three 150-ohm $\frac{1}{2}$-watt composition resistors for the 50-ohm internal loads. This combination was found to yield a very low reactance-to-resistance ratio at the operating frequencies. Standard General Radio 50-ohm loads were used to terminate the unused terminals when measuring VSWR and isolation characteristics. The measured isolation between outputs is plotted in Fig. 5 over the frequency range 450 to 550 mc. It will be noted, that at the center frequency of 500 mc the minimum isolation is 27 db, although it is not constant for all output terminals—a fact contrary to the theory based on the model of Fig. 3. The output VSWR is plotted in Fig. 6 and is about 1.6 at 500 mc, where the input VSWR is about 1.2. The output-to-input ratio was measured and found to be -9 db at all frequencies, indicating that the device was behaving as an 8-to-1 lossless power divider with negligible insertion loss.

The experimental model described above differs in several respects from the theoretical model. First, the theoretical model assumes a diameter which is negligible in wavelengths. In the experimental model it was 0.1λ. The principle effect of this appears to be that it shifts the center frequency for best performance to a lower value than the design frequency, and, perhaps, changes the effective resistance of the internal loads, since the current throughout the length of the resistors may no longer be uniform. Second, measurement of the phase between outputs showed several degrees of phase inequality, indicating that the perfect symmetry assumed in the theory had not been achieved in the splined structure and internal loads. Finally, the exact value of the characteristic impedance of the spline-to-shield transmission lines was probably not exactly $\sqrt{8}R_0$. Although the effects mentioned above can be overcome with further development work, the inequality of isolation between output terminals is believed to be due to a more fundamental reason. It will be noted that, shunting any pair of output terminals, there is a short circuited two-wire line formed by the pair of splines leading to these terminals. When the length of the splines is $\lambda/4$, an infinite impedance is shunted between output terminals; but at frequencies off the center frequency, a finite reactance appears between output terminals which is directly proportional to the characteristic impedance of the two-wire line formed by the splines. Because of the circular arrangement of output terminals, diametrically opposite splines (terminals 1–5 in Fig. 5), for example, would form a transmission line of higher characteristic impedance than adjacent splines (terminals 1–2 in Fig. 5). Since, the shunting effect is difference for each pair of output terminals, it is to be expected that the isolation will not be uniform for all terminals, except at the center frequency where there is no shunting effect for any terminal.